

interpolation between the values $B(r)$, where $i^2 = -1$

$$\theta(w) = (pe^{wi} + q)^s = \sum_{r=0}^{r=s} B(r)e^{rwi}.$$

In terminology of Laplace, $\theta(w)$ is the generating function of the sequence $B(r)$.

We shall first show that $B_0(x) = B(m)$ when x is a positive integer m . To prove this, substitute $\theta(w)$ from (1) and integrate. This gives

$$\sum_{r=0}^{r=s} B(r) \frac{\sin(r-x)\pi}{(r-x)\pi} \\ B(0) \frac{\sin(-x\pi)}{-x\pi} + B(1) \frac{\sin(1-x)\pi}{(1-x)\pi} \\ + \dots + B(s) \frac{\sin(s-x)\pi}{(s-x)\pi}.$$

When $x = m$ is a positive integer, each term but one of the series vanishes and this one has the value $B(m)$. Accordingly, $B_0(m) = B(m)$.

Formula (1) gives exactly the terms of the expansion of $(p+q)^s$ for positive integral values $x = m$. It may be considered an interpolation formula for values of x between the integral values.

We shall be interested in two developments of this interpolation formula. The first is based on the development of $\log \theta(w)$ in powers of w , and the second on the development in powers of p . The resulting types of development are known as the Type A and Type B series, respectively.